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Shock waves and cosmological matrix models

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Abstract: We find the shock wave solutions in a class of cosmological backgrounds with a null singularity, each of these backgrounds admits a matrix description. A shock wave solution breaks all supersymmetry meanwhile indicates that the interaction between two static D0-branes cancel, thus provides basic evidence for the matrix description. The probe action of a D0-brane in the background of another suggests that the usual perturbative expansion of matrix model breaks down.

Keywords: M(atrix) Theories.

It is an important open problem to formulate string/M theory in a time-dependent background in general, and to study what happens to cosmological singularity in particular. The recently proposed matrix description [1] of a cosmological background with a null singularity appears to be an advance forward, this construction was generalized to a class of backgrounds in [2]. An earlier example of cosmology with a null singularity is proposed in [3, 4] and is later studied by many authors [5]-[7], they find that this singularity is highly unstable due to gravitational back-reaction. It seems to us that models studied in [1, 2] and in the present note are devoid of such problems.

A cosmological background with a null singularity is not necessarily stable, even when it admits a perturbative string description [5]-[7]. It was found that a smeared shock wave changes the nature of the singularity, this happens in $2+1$ dimensions. Our solution presented below is also smeared in one or more directions, but localized in other directions, there is no sign of instability.

Matrix theory [8, 9] description predicts that there is no interaction between two static D0-branes, this is verified by our solution. The matrix action does not possesses linear supersymmetry (there is always non-linearly realized supersymmetry corresponding to shift of fermions), we show that the shock wave solution breaks all supersymmetry, in accordance with the matrix action prediction. Does the fact that there is no net force between two static D0-branes imply that we can expand the effective action in terms of relative velocity? The probe action in the background of a shock wave shows that this is not the case. This has far-reaching consequence: the usual perturbative expansion in matrix model breaks down.

The background studied in [1] is a flat 10D space-time with a linear dilaton proportional to the light-cone coordinate x^+. Its 11 dimensional M theory matric reads

$$ds_{11}^2 = e^{2Qx^+/3} ds_{10}^2 + e^{-4Qx^+/3}(dY)^2 \,. \tag{1}$$

The 10 dimensional part shrinks when $x^+ \to -\infty$, thus there is a null singularity a finite distance away in terms of the affine parameter. It is argued in [1] that the matrix model works for this background with a time dependent action, where the matrix time t is chosen to coincide with x^+.

The above solution can be generalized to the following class [2]

$$ds^2 = e^{2\alpha x^+}(-2dx^+dx^- + (dx^i)^2) + e^{2\beta x^+}(dx^a)^2 i = 1, \ldots, 9-d \,, \qquad a = 9-d+1, \ldots, 9 \,, \tag{2}$$

where the transverse directions are grouped into two sets. There are other generalizations in [2], but here we restrict our discussion to the above two cases.

It is believed that for each of these backgrounds there is a matrix description, which may provide a theory for resolving the singularity. For this claim to be valid, it is important to check whether these backgrounds are stable. Here we take an approach similar to that of [5]. Consider there be some gravitational sources, localized in x^- dimension as well as in the transverse dimensions. The existence of such sources will change the background. We want to see how large the change can be. First we consider the easiest case, with

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the background metric given by (1): It is convenient to treat the x^- and Y dimension as periodically identified,i.e.

$$x^- \sim x^- + 2\pi R\,, \qquad Y \sim Y + 2\pi R'$$

Consider the perturbation represented by stress tensor

$$T_{--} = \sum_a p_{-a} e^{-2Qx^+} \delta(\vec{x}_\perp - \vec{x}_{\perp a})\,, \tag{3}$$

where the x^+ dependence is introduced by energy conservation (this is for the background (1); for (2), the x^+ dependent factor is $\exp[-((9-d)\alpha + d\beta)x^+])$, and $x_\perp$ represents all the uncompactified transverse dimensions. The above stress tensor is due to multi-sources, located at $x_{\perp a}$ in the transverse directions. The stress tensor is smeared in the x^- and Y directions, thus p_{-a} is not really the longitudinal momentum of the a-th particle, the longitudinal momentum should be $p_{-a}2\pi R 2\pi R'$, where R is the radius of x^- and R' the radius of Y. We make the ansatz that the metric takes the following form

$$ds^2 = -2e^{2\alpha x^+} dx^+ dx^- - 2e^{\theta x^+} f(x^-, x^A)(dx^-)^2 + e^{2\alpha_A x^+}(dx^A)^2, \tag{4}$$

where $A = i, a$ in (2), θ is a constant to be determined. This ansatz can solve Einstein equations with source (3), and requires more non-vanishing components in stress tensor if the source is not smeared in direction Y (or in directions x^a), as we shall see shortly. The component g_{--} reflects the effect the source has on the geometry. If g_{--} vanishes, we get back to the original metric we started with, which should satisfy the vacuum Einstein equations with the constraint on the parameters:

$$\sum_A \alpha_A(\alpha_A - 2\alpha) = 0. \tag{5}$$

For metric (1), we have

$$\alpha = \alpha_i = \frac{Q}{3}\,, \qquad i = 1 \ldots 8\,; \qquad \alpha_9 = -\frac{2Q}{3}\,. \tag{6}$$

In the case of (2), there are

$$\alpha_i = \alpha\,, \qquad \alpha_a = \left(1 \pm \frac{3}{\sqrt{d}}\right)\alpha\,. \tag{7}$$

To find a solution with a source (3), we use the orthonormal basis

$$\begin{aligned} e^- &= dx^- \\ e^+ &= e^{2\alpha x^+} dx^+ + e^{\theta x^+} dx^- e^A = e^{\alpha_A x^+} dx^i\,. \end{aligned} \tag{8}$$

Up to symmetry, the non-vanishing components of spin connection are

$$\begin{aligned} \omega^+{}_+ &= \theta e^{(\theta - 2\alpha)x^+} dx^- \\ \omega^+{}_A &= e^{(\theta - \alpha_A)x^+} \partial_A f dx^- - \alpha_A e^{(\alpha_A + \theta - 2\alpha)x^+} f dx^+ \omega^A{}_+ = \alpha_A e^{(\alpha_A - 2\alpha)x^+} dx^A\,. \end{aligned} \tag{9}$$

The non-vanishing curvature 2-forms are

$$\begin{aligned}
\Omega^{+}{}_{+} &= \theta(\theta-2\alpha)e^{(\theta-4\alpha)x^+} f e^+ \wedge e^- + (\theta-\alpha_A)e^{(\theta-2\alpha-\alpha_A)x^+}\partial_A f e^i \wedge e^-,\\
\Omega^{+}{}_{A} &= \left[e^{(\theta-\alpha_A-\alpha_A)x^+}\partial_A\partial_B f - \alpha_A(\alpha_A-2\alpha)e^{(2\theta-4\alpha)x^+} f^2\delta_{AB} + \alpha_A e^{(\theta-2\alpha)x^+}\partial_- f\delta_{AB}\right]\times\\
&\quad \times e^B\wedge e^- + (\theta-\alpha_A)e^{(\theta-2\alpha-\alpha_A)x^+}\partial_A f e^+\wedge e^- + \alpha_A e^{(\theta-2\alpha-\alpha_B)x^+}\partial_B f e^A \wedge e^B +\\
&\quad +\alpha_A(\alpha_A+\theta-2\alpha)e^{(\theta-4\alpha)x^+} f e^A\wedge e^+,\\
\Omega^{A}{}_{+} &= -\alpha_A(\alpha_A-2\alpha)e^{-4\alpha x^+} f\ e^+\wedge e^- + \alpha^A(\theta+\alpha^A-2\alpha)e^{(\theta-4\alpha)x^+} f\ e^A\wedge e^-\,,\\
\Omega^{A}{}_{B} &= \alpha_A e^{(\theta+\alpha_A-2\alpha)x^+}e^A\wedge e^- - \alpha_B e^{(\theta+\alpha_B-2\alpha)x^+}e^B\wedge e^- - 2\alpha_A\alpha_B e^{(\theta-4\alpha)x^+}e^A\wedge e^B
\end{aligned} \tag{10}$$

In the Lorentz frame, the non-vanishing components of the Ricci tensor are

$$\begin{aligned}
R_{++} &= -\sum_i \alpha_A(\alpha_A-2\alpha)e^{-4\alpha x^+}\,,\\
R_{--} &= -\sum_A \alpha_A(\alpha_A-2\alpha)e^{(2\theta-4\alpha)x^+}\ f^2 + \sum_A e^{(\theta-2\alpha_A)x^+}\partial_A\partial_A f + \sum_A \alpha_A e^{(\theta-2\alpha_A)x^+}\partial_- f\,,\\
R_{+-} &= \left[\theta(\theta-2\alpha) + \sum_A \alpha_A(\theta+\alpha_A-2\alpha)\right]e^{(\theta-4\alpha)x^+} f\,,\\
R_{A+} &= 0,\\
R_{A-} &= \left(\theta - 2\alpha_A + \sum_B \alpha_B\right)e^{(\theta-2\alpha-\alpha_A)x^+}\partial_A f\,,\\
R_{AB} &= -2\alpha_A\left(\theta + \sum_C \alpha_C - 2\alpha\right)e^{(\theta-4\alpha)x^+} f\delta_{AB}\,.
\end{aligned} \tag{11}$$

Because the transformation from the orthonarmal basis to the coordinate basis does not mix the spatial spatial components, we see from (11) that the condition $T_{AB}=0$ implies

$$\theta - 2\alpha + \sum_A \alpha_A = 0\,. \tag{12}$$

This condition also implies that $R_{A-}=0$ if $\alpha_A=\alpha$. This is the case for the background (2) when $A=i$. If $\alpha_A\neq\alpha$, we will have to postulate $\partial_A f=0$, namely we need to smear the source in these directions. If we do not, a non-vanishing component T_{A-} is required, this is just fine, since R_{A-} thus T_{A-} is a total derivative, the component of momentum

$$p_A \sim \int T_{A-}\sqrt{g_\perp}dx^- d^9x \tag{13}$$

automatically vanishes. However, we will assume that as long as $\alpha_A\neq\alpha$, the source is smeared in direction x^A. Finally, Taking (12) as well as (5) into account, and changing back to the coordinate basis, we find the only non-vanishing component of Riccci tensor R_{--}

$$R_{--} = \sum_A e^{(\theta-2\alpha_A)x^+}\partial_A\partial_A f + \sum_A \alpha_A e^{(\theta-2\alpha_A)x^+}\partial_- f\,. \tag{14}$$

From now on we focus on the two cases (1) and (2) only. In the case of (1), f is independent of x^- and Y, and

$$\theta = -\frac{4Q}{3}\,. \tag{15}$$

The remaining Einstein equation is

$$8e^{-2Qx^+}\Delta f = {\kappa_{11}}^2 \sum_a p_{-a} e^{-2Qx^+} \delta(\vec{x}_\perp - \vec{x}_{\perp a}) \,, \tag{16}$$

where Δ is the Laplace operator in the 8 transverse dimensions. Finally we get the solution

$$f = -\sum_a \frac{\kappa_{11}^2 p_{-a}}{48 s_7 |\vec{x}_\perp - \vec{x}_{\perp a}|^6} \,, \tag{17}$$

where s_7 denotes the volume of the sphere S^7.

The surface defined by $x^+ =$ constant becomes spacelike now. So we will get a spacelike singularity. Quantitatively, when g_{--} and g_{+-} become the same order, the geometry is severely changed. Consider only one source, there is a radius:

$$r_0 = e^{-\alpha x^+} \left(\frac{48 s^7}{\kappa_{11} p_-} \right)^{\frac{1}{6}} . \tag{18}$$

The condition $r \gg r_0$ defines the asymptotical region. When $x^+ \to \infty, r_0 \to \infty$. It seems dangerous. But there is some subtlety here. First, When x^+ is finite, r_0 is also finite, and we do not need to worry about the change as long as we are far away from the source. As x^+ grows, r_0 vanishes, and so the future is changed little. Second, if we want to detect the change of background, we must introduce another observer. Here we put another graviton into the spacetime. The change of background is reflected by the change of action of the second graviton. As we will see later, this picture breaks down as $x^+ \to -\infty$. So even the nature of the singularity is changed, we cannot detect the change on the classical level. To further understand it, perhaps we should go to matrix theory. It is important to ask whether matrix theory is right here. In matrix theory, two gravitons correspond to block diagonal parts of a matrix. They interact via the off diagonal part. Spacetime emerges as we integrate out the off diagonal part. When $x^+ \to -\infty$, we reach the singularity in sugra, and a week coupling super Yang-Mills theory in matrix. As argued by [1], simply integrating out is not allowed. We must deal with the whole non-abelian degrees of freedom. To us, there is no sign of breakdown of matrix theory. More rigorous matrix calculation is needed to say anything more.

In a flat background as well as in a anti-de Sitter background, stability of a perturbation amounts to requiring that the space-time remains asymptotically flat or anti-de Sitter, since definition of S-matrix or boundary correlators demands this. In a time-dependent background with a big-bang singularity, intuitively, the space-time structure is required to remain the same where there is no singularity as time approaches infinity, since we may invoke perturbative string picture there. However, at the point where there is singularity, the definition of observables is not clear yet, so we do not know how to define stability. Nevertheless, the kind of divergence found in [6] is not allowed, happily that does not occur here.

Unlike the case discussed in [5], the change of the geometry is localized here. In the region far away from the sources, space-time is almost the same as the original background. In this sense, the background is stable. In the case of (2), the calculation is similar. Provided that we compactify x^- and x^a and smear the source in these directions, all components of Ricci tensor vanish except R_{--}. The solution of f is also similar, r^6 in the denominator in (17) is to be replaced by r^{7-d}. If we take the radius of x^- to infinity, the stress tensor should also be localized in this direction, the source will contain a factor $\delta(x^-)$, so does the component g_{--} of the metric. $\delta(x^-)$ is also localized, it will not trigger instability.

What we have obtained is noting other than the smeared shock wave representing the classical background of multi D0-branes. Eq. (17) clearly indicates that there is no net interaction among D0-branes. We now discuss whether this background still preserves supersymmetry.

Supersymmetry. In 11 dimensions, the only SUSY transformation of interest is that for the gravitino $\delta\psi_\mu = D_\mu\epsilon$. The components of spin connection of interest are ω_- and ω_i:

$$\begin{aligned}\omega_- &= 2\theta e^{(\theta-2\alpha)x^+} f\Gamma^{+-} + 2\sum_i e^{(\theta-\alpha_A)x+}\partial_A f\Gamma^{A-}\,,\\ \omega_A &= 2\alpha_A e^{(\alpha_A-2\alpha)x^+}\Gamma^{A+} - 2\alpha_A e^{(\theta+\alpha_A-2\alpha)x^+} f\;\Gamma^{A-}\,.\end{aligned} \tag{19}$$

Then the condition $D_+\epsilon = 0$ says that ϵ is independent of x_+. For the condition

$$D_-\epsilon = \left(\partial_- + \frac{1}{4}\omega_-\right)\epsilon = 0 \tag{20}$$

multiplied by Γ^-, it becomes

$$\left(\partial_- + \frac{1}{2}\theta e^{(\theta-2\alpha)x^+} f\right)\Gamma^-\epsilon = 0\,. \tag{21}$$

The second term is dependent of x^+, while the first term not. Thus they both vanish, implying that ϵ is also independent of x^-, and that ϵ is subjected to the constraint $\Gamma_-\epsilon = 0$. Similarly, multiply $D_A\epsilon = 0$ by Γ_-, we end up with $\Gamma_A\epsilon = 0$ for all transverse directions, ϵ has to vanish. Hence we have shown that the existence of sources breaks all supersymmetry.

Interaction. We now switch to the convention in which $x^- = x - t$, hence the momentum p_- is always positive, and f is always negative. Consider the case that there is only one source. The solution looks much the same as the Aichelburg-Sexl metric. Hence we follow the approach of [10] to discuss the probe action of graviton of momentum N_2/R in the background of graviton of momentum N_1/R. When N_1 is large, the first graviton is treated as a classical source and produces the metric we have got. Then take the other particle as a probe particle. To get the action of the probe graviton in this background, the trick is to begin with the action for a massive scalar in eleven dimensions

$$\begin{aligned}S &= -m\int d\tau\,(-G_{\mu\nu}\dot{x}^\mu\dot{x}^\nu)^{\frac{1}{2}}\\ &= -m\int d\tau\left(-2e^{2\alpha x^+}\dot{x}^- - \sum_A e^{2\alpha_A x^+}(\dot{x}^A)^2 + 2e^{\theta x^+} f\dot{x}^-\dot{x}^-\right)^{\frac{1}{2}}.\end{aligned} \tag{22}$$

A dot denotes ∂_τ. Carry out a Legende transformation on x^-:

$$p_- = m\left(e^{2\alpha x^+} - 2e^{\theta x^+} f\dot{x}^-\right)\left(2e^{2\alpha x^+}\dot{x}^- - e^{2\alpha_A x^+} v_A^2 + 2e^{\theta x^+} f\dot{x}^-\dot{x}^-\right)^{-\frac{1}{2}}. \tag{23}$$

In our case p_- is to be fixed. Taking the limit $m \to 0$ implies that $G_{\mu\nu}\dot{x}^\mu\dot{x}^\nu \to 0$. Then

$$\dot{x}^- = \frac{e^{2\alpha x^+} - \sqrt{e^{4\alpha x^+} + 2\sum_A e^{(\theta+2\alpha_A)x^+}(\dot{x}^A)^2}}{2fe^{\theta x^+}}. \tag{24}$$

The appropriate lagrangian is the Routhian,

$$\mathcal{L}'(p_-) = -\mathcal{R}(p_-) = \mathcal{L} - p_-\dot{x}^-(p_-). \tag{25}$$

When $m \to 0$ with fixed p_-,

$$\begin{aligned} \mathcal{L}'(p_-) &\to -p_-\dot{x}^- \qquad (26)\\ &= p_-\left\{\frac{1}{2}\sum_A e^{2(\alpha_A-\alpha)x^+}\left(\dot{x}^A\right)^2 - \frac{1}{4}e^{-6\alpha+\theta}\left[\sum_A e^{2\alpha_A x^+}\left(\dot{x}^A\right)^2\right]^2 f + O\left(f^2\dot{x}_\perp^4\right)\right\}. \end{aligned}$$

The interaction terms all involve relative velocity v. If the source and the probe stay relatively static in the transverse space, there is no net interaction. This has to be the case if matrix model is correct, since two static D0-branes form a solution in matrix model, and induces no interaction terms in the matrix model action.

When the parameters take the values in (6), we get

$$\begin{aligned} \mathcal{L}'(p_-) \to p_-\Bigg\{&\frac{1}{2}[\sum_i^8(\dot{x}^i)^2 + e^{-2Qx^+}(\dot{x}^9)^2] - \\ &-\frac{1}{4}\left[\sum_i^8 e^{-Qx^+}(\dot{x}^i)^2 + e^{-3Qx^+}(\dot{x}^9)^2\right]^2 f + O\left(f^2\dot{x}_\perp^4\right)\Bigg\}. \end{aligned} \tag{27}$$

We see that as $x^+ \to -\infty$, those "higher order" terms become larger and larger, it is a bad idea to do expansion in terms of the transverse velocity.

The double expansion in v and r^{-1} corresponds to perturbative expansion in the matrix model, the breaking-down of this double expansion implies that even when one is treating interaction between two D0-branes, one should no do perturbative calculation in matrix model. It is rather curious that this breaking-down happens, since the matrix model or the matrix string theory in question becomes weakly coupled at early times. Our interpretation of this breaking-down is the following. Although the theory becomes weakly-coupled, the picture of two separated D0-branes becomes worse and worse at early times, since the off-diagonal modes become lighter and lighter, this can be seen for instance from the interaction term

$$Rg_s^{-2}\,\mathrm{tr}[X^i, X^j]^2 = \mathrm{R}e^{2Qt}\,\mathrm{tr}\left[X^i, X^j\right]^2, \tag{28}$$

where $t = x^+$. The mass of the off-diagonal modes is proportional to e^{Qt}. The breaking-down of velocity expansion is simply due to appearance of new light modes. Thus, although the D0-brane "moduli" (position) description breaks down, the matrix model does not, one simply has to deal with all non-abelian degrees of freedom. This is how matrix model resolves the singularity.

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